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## REQUIREMENTS TO A RADIO-FREQUENCY SPECTRUM OCCUPANCY EVALYATION

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Measurement of a radio-frequency spectrum occupancy is necessary for efficient management of this resource and should be carried out with a given accuracy and confidence [1]. In documents [2-4] recommendations are given concerning provision of required quality of measurements; however Spectrum Monitoring Handbook [2] and Recommendation [3] are based on limitation of a permissible relative error of measurements, and Report [4] is based on limitation of an permissible absolute error. Each of these approaches has their advantages and disadvantages. The present work is aimed on development of recommendations for choice requirements to estimation accuracy of the spectrum occupancy which would combine advantages of these two approaches.

Studying of documents [1-4] shows, that accuracy and confidence of occupancy estimations depends on a number of data samples obtained from a monitoring equipment during data integration interval $T_{I}$, that usually equals to 5 or 15 minutes, from a degree of occupancy of analyzed radio channel and from character of transmitted signals.

Let's analyze, in the beginning, a case of an occupancy estimation in a channel with pulse signals whose durations are less than one part per thousand from duration of interval $T_{I}$. Taking into account the expression [5]:

$$
\begin{equation*}
J_{m}=S O \cdot(1-S O) \cdot\left(x_{P} / \Delta_{S O}\right)^{2}, \tag{1}
\end{equation*}
$$

where: $J_{m^{-}}$a number of data samples, $S O$ - a channel occupancy degree, $x_{p}$ - is a percentage point of the probability integral that corresponds to the confidence level $P_{S O}, \Delta_{S O}$ an absolute measurement error,
it is possible to write for such signals the following relation between a number of accumulated samples $J_{m}$ and a provided absolute estimation error $\Delta_{S O}$ :

$$
\begin{equation*}
\Delta_{S O}=x_{P} \cdot \sqrt{S O \cdot(1-S O) / J_{m}} \tag{2}
\end{equation*}
$$

On the basis of (2), Fig. 1 shows dependences of the relative $\delta_{S O}=\Delta_{S O} / S O$ (continuous lines) and absolute $\Delta_{\text {SO }}$ (dotted lines) errors of an occupancy estimations for pulse signals from a number of samples $J_{m}$ at different values of the occupancy $S O$ and the confidence level of $95 \%$. Square-law character on $S O$ of a radicand in (2) shows, that under fixed $J_{m}$ for any couple of occupancy values $S O^{*}$ and $\left(1-S O^{*}\right)$ the absolute error of measurements appears to be identical, and that is the reason of two-valuedness of function $\Delta_{S O}\left(J_{m}\right)$. The greatest deflection to the right of dotted lines showing fixed absolute errors is indicated at occupancy values of $S O=50 \%$, pointing out that exactly at such occupancy values the greatest numbers of data samples $J_{m}$ are required for carrying out accurate and authentic measurements.

Fig. 1 visually demonstrates essential differences in character of variations of relative and absolute errors of occupancy estimations. The same differences in some other


Figure 1. Dependence of the relative $\delta_{S O}$ and absolute $\Delta_{S O}$ errors of occupancy estimations for pulse signals from the amount of samples Jm and different values of the occupancy $S O$ under the confidence level of $95 \%$
form demonstrate table 1 which shows numbers of independent samples that are required to provide measurements under various occupancy values with accuracies equal to $10 \%$ of a relative error or $0.5 \%$ of an absolute error at a confidence level of $95 \%$. From table 1 it follows that fixing requirements by only to a relative or only to an absolute error leads to a sharp increasing of the necessary data sample numbers for channels with high occupancy, or for low occupied channels. The analysis of these data and the recommendations containing in documents [2-4], allows to come to the following conclusions.

Table 1

| Occupancy, \% | Relative error of $10 \%$ |  | Absolute error of $0.5 \%$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Resultant value of <br> an absolute error, <br> $\%$ | Number of <br> necessary <br> independent <br> samples | Resultant value of <br> a relative error, $\%$ | Number of <br> necessary <br> independent <br> samples |
| 1 | 0.1 | 38047 | 50 | 1522 |
| 2 | 0.2 | 18832 | 25 | 3013 |
| 3 | 0.3 | 12426 | 16.7 | 4473 |
| 4 | 0.4 | 9224 | 12.5 | 5903 |
| 5 | 0.5 | 7302 | 10 | 7302 |
| 10 | 1.0 | 3461 | 5 | 13835 |
| 15 | 1.5 | 2178 | 3.3 | 19600 |
| 20 | 2.0 | 1537 | 2.5 | 24596 |
| 30 | 3.0 | 897 | 1,7 | 32283 |
| 40 | 4.0 | 576 | 1.25 | 36894 |
| 50 | 5.0 | 384 | 1.0 | 38432 |
| 60 | 6.0 | 256 | 0.83 | 36894 |
| 70 | 7.0 | 165 | 0.71 | 32283 |
| 80 | 8.0 | 96 | 0.62 | 24596 |
| 90 | 9.0 |  | 0.56 | 13835 |

Limitation of a permissible relative error of measurements by the value of $10 \%$, offered by the Handbook [2] and the Recommendation [3], means that at the actual occupancy of channels exceeding $90 \%$, a permissible absolute error of the occupancy estimation achieves $\pm 9 \%$. It allows at data gathering step managing with only several hundreds of samples, but it corresponds to a rather rough estimation as it 4 concerns values of an absolute error. For channels with occupancy less than $5 \%$ it, on the contrary, leads to absolute error values to be less than $\pm 0.5 \%$ that is possible only at accumulation more
than $10^{4}$ data samples, i.e. considerable expenses of hardware operational power are necessary.

Limiting, in accordance with Report [4], of an absolute error by value of $0.5 \%$ for low occupied channels appears to be a rational one. Though in channels with occupancy less than $5 \%$ the considerable relative error can be observed the fact that real deviations are not beyond $\pm 0.5 \%$ testifies to an adequate accuracy of measurements. So, for an occupancy equals to $1 \%$ the margins of an estimation confidential interval between $0.5 \%$ and $1.5 \%$ all the same characterize a rather small occupancy, and hardly it is expedient to waste time on obtaining a big number of additional data samples for acknowledgement of this obvious fact with some greater accuracy expressed in the tenth parts of a percent. At the same time, for channels with the occupancy exceeding $10 \%$ the permissible absolute error only in $\pm 0.5 \%$ establishes unduly rigid requirements to an estimation error and to the calculation resources to be spent.

Evaluating the data given above, and being based on a practical operational experience of spectrum monitoring services, it is possible to formulate the following recommendations. For the radio channels characterized by the occupancy not exceeding $5 \%$, it is expedient to establish requirements to a permissible absolute error of estimations at the level between 0.5-1 \% that guarantees sufficient accuracy of spectrum occupancy estimations under reasonable expenses of hardware operational resources. For channels with occupancy more than $10 \%$ both considered above options of accuracy limiting appear to be far from an ideal since in channels with high occupancy at the fixed relative error occupancy estimations happen to be a rough enough, and provision of the small fixed absolute error demands excessive hardware expenses. Thus, the most rational appears to be the intermediate option at which with increasing of the spectrum occupancy values the requirements to a relative error become tougher under insignificant increase of an absolute error of estimations.

As examples of similar effective compromise requirements the following two rules defining a permissible absolute error of occupancy estimations can be proposed:
the linear rule

$$
\begin{equation*}
\Delta_{S O}=0.005+0.05 \cdot S O \tag{3}
\end{equation*}
$$

and "convex" rule

$$
\begin{equation*}
\Delta_{S O}=0.0016+0.02 \cdot \sqrt{S O \cdot(1.86-S O)} . \tag{4}
\end{equation*}
$$

Relevant values of permissible absolute and relative errors of estimations calculated by (3) and (4), and corresponding values of the necessary sample numbers are presented in table 2. Its analysis shows the following:

1. Though rules (3) - (4) in low occupied channels accept relative error values that exceed the requirements of the Recommendation [3], these errors in an absolute domain
are rather small and for channels with occupancy less than $5 \%$ equal no more than $\pm 0.7 \%$. Rejection of excessively rigid requirements to occupancy estimation accuracy given in the Recommendation [3], allows to reduce the number of the necessary samples at least in 3 times.
2. For channels with occupancy more than $50 \%$ the rule (3) and, especially, the rule (4) lead to the necessity of essentially increase sample numbers in comparison to requirements of the Recommendation [3]. However on an absolute value this number of samples does not exceed $85 \%$ from the maximal one (see table 2 for channels with the occupancy from 15 to $20 \%$ ) for the rule (4) and $30 \%$ from the maximal one for the rule (3). With that, the estimation accuracy in comparison with the Recommendation [3] is magnified on $60 \ldots 80 \%$ for a rule (3) and in $2.7 \ldots 4.5$ times for a rule (4).

Table 2

| Occupancy, \% | Linear rule (3) |  |  | "Convex" rule (4) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Required absolute error, \% | Required relative error,\% | Necessary sample number | Required absolute error, \% | Required relative error,\% | Necessary sample number |
| 1 | 0.55 | 55.0 | 1258 | 0.43 | 43.2 | 2038 |
| 2 | 0.60 | 30.0 | 2092 | 0.54 | 27.2 | 2548 |
| 3 | 0.65 | 21.7 | 2647 | 0.63 | 21.0 | 2830 |
| 4 | 0.70 | 17.5 | 3012 | 0.70 | 17.5 | 3015 |
| 5 | 0.75 | 15.0 | 3245 | 0.76 | 15.2 | 3147 |
| 10 | 1.00 | 10.0 | 3461 | 1.00 | 10.0 | 3461 |
| 15 | 1.25 | 8.3 | 3136 | 1.17 | 7.8 | 3562 |
| 20 | 1.50 | 7.5 | 2733 | 1.31 | 6.6 | 3570 |
| 30 | 2.00 | 6.7 | 2018 | 1.53 | 5.1 | 3456 |
| 40 | 2.50 | 6.3 | 1476 | 1.69 | 4.2 | 3236 |
| 50 | 3.00 | 6.0 | 1068 | 1.81 | 3.6 | 2935 |
| 60 | 3.50 | 5.8 | 753 | 1.90 | 3.2 | 2558 |
| 70 | 4.00 | 5.7 | 504 | 1.96 | 2.8 | 2096 |
| 80 | 4.50 | 5.6 | 304 | 2.00 | 2.5 | 1535 |
| 90 | 5.00 | 5.6 | 138 | 2.02 | 2.2 | 848 |

As it follows from table 2, the absolute values of sample amounts do not exceed $85 \%$ from the maximal one (see table 2 for channels with the occupancy from 15 to $20 \%$ ) for the rule (4) and $30 \%$ from the maximal one for the rule (3). Therefore, the "convex" rule of calculation of a permissible absolute occupancy estimation error in relation to the linear one not only provides an appreciable increasing of the accuracy but also simultaneously
stabilizes requirements to sample amounts used for occupancy estimations. This fact, and also convenience of use in practice of the fixed sample amounts at the data gathering process, proves usefulness of analysis of errors variations under condition of $J_{m}=$ const. Results of such study for pulse signals and cases $J_{m}=3600$ and $J_{m}=1800$ samples are presented by continuous lines in Fig. 2 which shows the dependence of absolute errors $\Delta_{S O}$ of spectrum occupancy estimations obtained with the confidence level $95 \%$ under the fixed amount of data samples. Fig. 2 shows that the occupancy estimations counted on 3600 samples possess high accuracy features. Moreover, even at the ability of monitoring equipment to process only with 1800 samples during an interval of data accumulation the error of obtained occupancy estimations appears to be quite sufficient for purposes of spectrum monitoring:

- For radio channels with occupancy from 25 to 75 percent, having only on pulse signals, real occupancy of a spectrum can differ from the estimations received at 1800 elections of data, approximately on $\pm 2 \%$ that there correspond to a relative error no more than $8 \%$;
- At $S O<18 \%$ the relative error of estimations of occupancy is more than value in 10 $\%$, offered by the Recommendation [3], however on an absolute value it becomes not too great and makes no more $\pm 1.5 \%$ for channels with occupancy $S O \leq 12 \%$ and no more $\pm 1.0 \%$ for channels with the occupancy $S O<5 \%$.


Figure 2. Absolute error $\Delta_{S O}$ of spectrum occupancy estimations obtained with the confidence level $95 \%$ under the fixed amount of data samples $J_{m}$

It is necessary to recognise, that though for channels with $S O \geq 10 \%$ accuracy of the estimations received on 1800 samples, it appears quite satisfactory for purposes of spectrum monitoring. With reference to channels with the occupancy $\mathrm{SO}=5 \ldots .10 \%$ the measurements accuracy provided by $J m=1800$ appears, perhaps, to be a minimum admissible. Thus, for channels with pulse signals it is necessary to use the equipment providing at least 1800 samples of a channel condition for during the data accumulation interval, i.e. a minimum of 2 samples per a second at $T_{I}=15$ minutes and 6 samples in a second at $T_{I}=5$ minutes.

Besides the data related to pulse signals, Fig. 2 presents, by dash-dotted and dashed lines, a set of data characterizing occupancy measurement accuracy of channels transmitting lengthy signals (whose durations $\tau_{s}$ considerably exceed $0.001 T_{I}$ ). These data are obtained in accordance with the procedure presented in [6] which states that at the time equidistant testing of a radio channel condition the absolute estimation error depends from a samples amount as:

$$
\begin{equation*}
\Delta_{Z}=\frac{x_{P}}{2 J_{m}} \sqrt{1.06 V_{a v r}} \tag{5}
\end{equation*}
$$

where $V_{a v r}$ is an average amount of lengthy signals acting in the channel during of the data accumulation interval. Distinction between two groups of curves presented by Fig. 2 for lengthy signals reflects difference of the reasons, which produce variation of the occupancy.

Dashed horizontal lines shown in Fig. 2 correspond to a case when the occupancy variation occurs at the expense of variation of duration of signals at their fixed average quantity. For example, for the line $V_{a v r}=100$, corresponding to existence in the channel of 100 signals during the data accumulation interval, occupancy in $50 \%$ is provided at $\tau_{s} / T_{I}=0.005$ under the same extent of spaces between signals, and occupancy in $25 \%$ turns out at reduction of the signal durations to $\tau_{s} / T_{I}=0.0025$ and increasing of spaces durations to $\tau_{p} / T_{I}=0.0075$.

If occupancy variations are caused by variations of average amounts of signals acting during a data accumulation interval under their invariable durations the character of error variations appears to be different. Let signal durations during the data accumulation interval remain the same and equal to $\tau_{s} / T_{I}=0.0025$, than the occupancy in $50 \%$ is provided at quantity of signals $V_{a v r}=50 \% /\left(\tau_{s} / T_{I}\right)=200$, and the occupancy in $25 \%$ turns out at quantity of signals $V_{a v r}=25 \% /\left(\tau_{s} / T_{I}\right)=100$. Corresponding curves for $J_{m}=600$ are shown in Fig. 2 by the set of dash-dotted lines.

From Fig. 2 it follows that even for rather short signals meeting the condition $\tau_{s} / T_{I}=0.0025$, already at $J_{m}=600$ the occupancy estimation error for $S O<50 \%$ appears to be less than that for pulse signals under $J_{m}=1800$. And only for the occupancy
$S O>75 \%$ the absolute estimation error under $J_{m}=600$ appears to be a big one - exceeds $3 \%$, however even in this case the relative estimation error remains lower than it is required according to the Recommendation [3]. It proves that for occupancy measurements in channels with lengthy signals it is possible to use the equipment providing 600 samples during the data accumulation interval.

The conclusion. Updating of requirements to accuracy of the occupancy estimation, defined by rules (3) and (4), provides comprehensible to spectrum monitoring practice a permissible occupancy estimation error in application to the whole range of its possible values and allows to carry out measurements at the hardware expenses corresponding to possibilities of a modern monitoring equipment. If the monitoring equipment does not allow to carry out a flexible tuning of accumulated sample numbers for different radio channels at a choice of a mode of its work is admissible to be guided by obtaining a minimum 600 samples during an integration interval for channels with lengthy signals, and not less than 1800 samples during an integration interval for other channels. These rates of data gathering guarantee comprehensible accuracy and confidence of occupancy measurements even for radio channels with pulse signals. For channels where there are no more than 250 lengthy signals during the data integration interval calculation of the occupancy estimations using 1800 samples provides an absolute estimation error less than $1 \%$ at the confidence level of $95 \%$.

Results of the presented analysis might be a subject of proposals on refining some provisions of the Recommendation [3].

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